MRI is a non-invasive medical imaging technique. It constructs images based on the proton’s signal responses to a train of radiofrequency excitations and pulsed magnetic field gradients (pulse sequence) under different chemical environments. The MRI signal decays exponentially with the tissue and field inhomogeneity-dependent time constant T2\* and the amount of T2\* decay of the signal acquired is determined by the choice of echo time (TE), the relationship can be described by the equation S(TE). As the excessive iron caused by PD distorts the magnetic field, a more rapid decay of the MRI signal and a smaller T2\* value will be observed at the SN, various studies have shown this change of T2\* value can be used as an effective biomarker for PD \cite{Intro5,Intro6}. A traditional way of generating quantitative T2\* images is to acquire MR images at different TEs and fit the signal’s exponential decay in each voxel to get the T2\*, this technique is known as T2\* mapping using a Multi-Echo Spoiled Gradient -Recalled (SPGR) sequence \cite{Intro7}. However, the complex tissue movement at the brain stem, which is related to blood flow, central cerebellum fluid flow, and the cardiac cycle, results in tissue motion artefacts on MR images and makes high-quality brain stem MRI difficult to achieve \cite{Intro8,Intro11}. Additionally, it makes getting an accurate T2\* image of the brain even more difficult as one T2\* image is constructed based on multiple high-quality MR images.\par \vspace{1em}

A simple way to reduce the effect of tissue motion is acquiring multiple MR images and averaging to reduce motion artefacts, but it will lengthen the acquisition time. Using a larger voxel size can also reduce the impact of the tissue movement, however, high-resolution MR images are essential to capture important tissue information from the tiny millimetre-size SN subregion nigrosomes, where PD-related changes first build up in SN \cite{Intro8}. Other post-processing-based methods can perform motion correction on the acquired images, but these techniques mostly focus on the rigid whole-body movement instead of the complex non-rigid tissue motion \cite{Intro9}.\par \vspace{1em}

Based on all these limitations, a new T2\* imaging technique is suggested \cite{Intro10}, which can reduce the impact of motion while maintaining short acquisition time and good resolution, using the MRI pulse sequence named k-space-aliased Spoiled Gradient-Recalled (ka-SPGR). The images acquired with ka-SPGR are not simply related by decay time, instead, each image contains a complex summation of signal from multiple echo times. By extracting each signal component of this complex sum, the T2\* related signal decay at specific echo times can be reconstructed and T2\* can be calculated by fitting the exponential decay. Theoretically, even if the movement of the brain creates artefacts in some acquisition, it will not have a huge impact on the reconstructed T2\* decay signal, as the signal is obtained by averaging across multiple acquisitions. However, so far there has not yet been any quantitative analysis of the ka-SPGR T2\* measurement accuracy compared to the standard multi-echo SPGR approach, and no suggested optimal MRI scanning parameters for the ka-SPGR sequence. Therefore, evaluating the ka-SPGR’s T2\* accuracy and obtaining optimal parameters is desirable, before examining the motion robustness of the ka-SPGR technique in vivo.\par \vspace{1em}

\subsection{N-periodic ka-SPGR}

The N-periodic ka-SPGR sequence is based on short TR fast SPGR \cite{Method4} with TE = TR/2.

The repeated excitation with short TR leads to the formation of a steady-state signal. Additionally, radiofrequency-spoiling (RF-spoiling) is applied by incrementing the RF excitation phase in a quadratic series, as shown in the equation below \cite{Method4, Intro10},

\begin{equation}

\phi\_n = 0.5 \times n^2 \times \frac{2\pi}{N} , (n = 0,1,2 ...)

\end{equation}

, where n is the index of the RF excitation, and N is the chosen periodicity. By using this quadratically changing excitation phase, the signal acquired after the RF excitation will periodically yield to N different steady-states S(0), S(1)…S(N-1). Each steady-state signal is the summation of the T2\* related decay signals at specific times ($Ae^{-(TE+F\times TR)/T\_2^\*}$) weighted by corresponding phase modulation ( ), and the whole analytical solution of the steady-state signal after the nth RF pulse is shown below\cite{Intro10},

\begin{equation}

S(n) = e^{j\omega TE}\sum\_{F}^{} Ae^{-(TE+F\times TR)/T2^\*} e^{jF\omega TR} e^{jnF \frac{2\pi}{N}}.

\end{equation}

, where omega is the off-resonance frequency, A is a constant, F is the F-state index, and the {} and {} terms relate to spin-off-resonance precession. The T2\* related component in the equation, $Ae^{-(TE+F\times TR)/T\_2^\*}$, is known as the configuration state or F-state signal and is denoted as $F\_0$ if F in the equation equals 0.

For a given acquisition, the user can choose how many F-state components are included in the measured signal by adjusting a spoiler gradient in the sequence. This shifts the different F-state signals apart from each other in k-space, as shown in Figure[]. Notice that the ideal case will be having exactly N F-states in the acquired signal.

The k-space of each F-state can be reconstructed by summing up N-acquired signals with corresponding phase modulation ( ) followed by shifting the k-space back to the centre. An inverse Fourier transform is then performed to generate the F-states images, which are then used to fit T2\*. An illustration of this procedure using 5-periodic ka-SPGR is provided in Figure \ref{Figure:Theory1}.